HIGH SCHOOL TEACHERS' COGNITIVE SCHEMES SHOWN IN PROBLEM SOLVING APPROACHES BASED ON THE USE OF TECHNOLOGY

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This study documents the type of proof schemes that high school teachers developed and used in problem solving scenarios that involve the use of dynamic software (Cabri-Geometry). Research questions that helped organize and structure the development of the study include: (i) To what extent does the high school teachers' process shown to pose questions or formulate problems influence their ways to validate mathematical relations or conjectures? (ii) What types of problem solving strategies do the participants use to identify and support conjectures that emerge as a result of constructing and examining dynamic problem representations? Results indicate that the subjects' use of dynamic software to represent mathematical objects and situations dynamically not only favors their ways to formulate conjectures; but also the schemes' construction to support and validate those conjectures.

How does a mathematical relation emerge? What does it mean to prove or demonstrate a particular mathematical relation? What types of arguments are important to validate a mathematical conjecture? How visual, empirical, geometric, and analytic arguments are used to validate mathematical relations? To what extent do the systematic use of dynamic software favor or enhance a particular ways of reasoning and thinking about proofs' construction? The discussion of these types of questions sheds light on the complexity involved during the subject's construction of mathematical arguments and the relevance of problem solving approaches that promote the teachers and students' use of technology to foster both the formulation of relations and the search for arguments to support mathematical conjectures. Those problem-solving experiences involve the subject's direct participation in formulation of questions or problem posing activities, the development of problems solving strategies and the use of different artifacts, including computational or digital tools, to represent and explore mathematical ideas or problems.

It is common to associate the term "mathematical proof" to the development and presentation of deductive arguments, based on a set of propositions, to support results or mathematical relations; however, the process of proving involves more than only the use of logic or formal arguments; it includes for the subject to be convince himself/herself initially and to convince others about the viability and validity of the conjecture or mathematical relation to be proved (Harel & Sowder, 1998). What does it mean for the subject to be convinced about the validity of a particular mathematical relation? We argue that it means the opportunity for the subject to explore conjectures or mathematical relations in terms of visual and empirical explications that often rely on measuring figures or attributes (areas, perimeters, lengths, etc.) and moving objects and observing patterns of particular behaviors. Since the use of technology seems to facility the representation and exploration of mathematical situations, then it is important to investigate the extent to which the use of particular tools helps teachers and students develop ways of reasoning that favor the use of distinct arguments to validate and prove mathematical conjectures or results.

Thus, in this study, we are interested in documenting and analyzing the process exhibited by high school teachers to construct dynamic representations of situations that lead them to

Lamberg, T., & Wiest, L. R. (Eds.). (2007). Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Stateline (Lake Tahoe), NV: University of Nevada, Reno.

formulate and examine conjectures and ways to support them. The research questions used to guide and structure the development of the study were:

1. To what extent does the high school teachers' processes shown to pose or formulate questions influence their ways to validate mathematical relations or conjectures? Here, there is interest to document how the participants' construction of dynamic representations of situations helped them to initially pose questions that eventually led them to identify and explore mathematical conjectures. Similarly, we focused on analyzing ways in which the participants look for arguments to support those conjectures. In particular, we identify and discuss the proof schemes that emerged through the participants' use of dynamic software.
2. What types of problem solving strategies do the participants use to identify and support conjectures that emerge as a result of constructing and examining dynamic problem representations? Here, we focused on documenting the types of problem solving strategies (examining particular cases, looking for patterns, using coordinate system, and finding objects' loci) used to solve problems and construct arguments and proofs.

Conceptual Framework

An important feature in the process of learning mathematics is the construction of a line of thinking in which the learners have the opportunity of using their previous knowledge to identify mathematical relations and to provide arguments to support results. Harel and Sowder (1998) distinguish two related aspects that are relevant during the subjects' construction of proofs or arguments to justify conjectures: The subject self-convincement stage in which he/she is convinced that the conjecture is valid and make sense to him/her; and the need to persuade others about the validity of that conjecture. That is, one is an individual recognition and the other a community acceptance. Harel and Sowder (1998) also identify seven types of sub-categories of proof schemes: (a) ritual proof scheme in which the subject's convincement is based on accepting the form rather than the content or argument; (b) authoritarian proof scheme in which the subject's convincement is based on arguments or affirmations presented by an authority (teacher, textbook, or expert); (c) symbolic proof scheme in which conviction is based on symbolic manipulations without explicit explanation of the meaning attached to those manipulations; (d) perceptual proof scheme in which conviction is based on using rudimentary mental images that lack actions to anticipate results; (e) inductive proof scheme in which conviction is achieved through the use of quantitative evaluations; (f) transformational proof scheme in which the subject relies on goal-oriented operations on objects to anticipated results; and (g) axiomatic proof scheme which is also a transformational proof that relies on the use of axioms and established definitions.

Thus, each proof scheme becomes relevant to explain the cognitive process embedded in both the subject's own convincement about the validity of mathematical relations and the subject process to convince others about the pertinence, meaning, and proof of that conjecture.

We also argue that the cognitive process involved during the construction of mathematical arguments to support relations can be traced or explained in terms of the subject's ways to formulate and pursue significant questions (Santos-Trigo, et al., in press). Thus, problem solving approaches that encourage students/learners to formulate, examine, and support conjectures might help them value the use of distinct types of arguments to justify results and conjectures. Santos-Trigo (2007) illustrates ways in which high school teachers and students can transform typical textbook problems into nonroutine problems when they construct various representations (including dynamic representations) of those problems and look for distinct ways to approach them. In particular, the use of computational tools (dynamic software for example) seems to offer proper conditions for the learners to pose and examine questions that lead them to formulate and later support conjectures. In this context, we are interested in documenting the extent to which the categories identified by Harel and Sowder (1998) can be used to explain the subjects' construction of arguments within a problem solving environment that promote the use of dynamic software.

Participants, Design, and Procedures

Seven high school teachers participated in a weekly 1.5 hr problem solving sessions during one semester. However, we focus on the work shown by three of those participants because their approaches to the tasks are representative of the group's work. The aim of the sessions was to work on a series of tasks that involves the construction of geometric configuration, using Cabri-Geometry software to identify and support mathematical relations or conjectures. In general, the pedagogic approach that consistently characterized the development of the sessions included:

1. The responsible or coordinator of the sessions introduced a task to the participants and explains to them ways to work and report their work.
2. The participants worked on each problem individually and later they had opportunity to present and examine their work within the group.
3. At the end, each participant handed in a report that included electronic files and written comments and observations that appeared during their individual and collective participation.

To analyze what the three participants showed during their problem solving approaches, we focus on those tasks that involve the construction of geometric configurations that were used to identify and discuss mathematical relations. The initial tasks and instructions that the participants received to construct those dynamic configurations included:

1. Given a line and a point that does not belong to that line, construct an isosceles triangle with one side lying on the line and the third vertex the given point that is not on that line.
2. Draw a square given one of its vertices, and the middle point of one of the side of the square that is not adjacent to the given vertex.
3. Draw a tangent circle to two given circles.

Data used to analyze the participants' approaches to the tasks come from electronic files, written reports, and field notes taken by the sessions' coordinator during the problem sessions. The first goal was to analyze the extent to which the proof schemes identified by Harel and Sowder (1998) consistently appear in the participants' performances. In addition, the identification of problem solving episodes (Schoenfeld, 1985) became important to identify the type of strategies used to identify, construct, and support mathematical relations.

Presentation of Results and Discussion

There is evidence that the use of the software became important for the participants to initially identify key elements which they used to construct a dynamic representation of the tasks. Thus, dragging particular points or objects within the representation was an important activity that helped them detect invariants or conjectures. For example, Ann approached the first task (isosceles triangle) by drawing a line l and point C out of that line. She chose point P on line l and drew a circle with center at point C and radius the segment CP. Thus the triangle PCQ is isosceles by construction. In this case, Ann observed that she could draw a family of isosceles triangles when point P is moved along the line l and asked at what position of P the triangle PCQ becomes equilateral? (Task 1.1) (Figure1a). Hugh drew line l and point Q on that line and a circle with center at point Q and radius QC (C is not on line l). Then he drew the perpendicular bisector of segment QC, located point S and asked: What is the locus of the point S when point Q is moved along the line? (Figure1b). He observed that the locus of point S was a line and verified this assertion by selecting two points on the locus and drawing a line passing by those points and observed that it overlaps the locus.

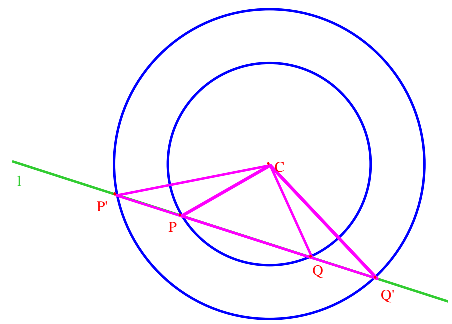

Figure 1a: At what position of point P, triangle PCQ becomes equilateral?

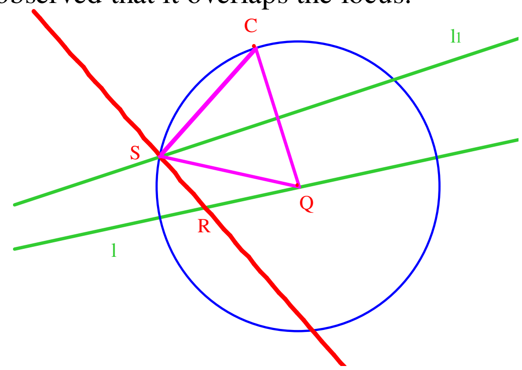

Figure 1b. What is the locus of the point S when point Q is moved along line l?

Hugh noticed that the locus intersects line l at point R (Figure 1b) and then he drew segment CR and a circle with center at point C and radius CR, and located point T to construct the equilateral triangle. He used the software to measure the angles in order to verify the measure of each interior angle was 60 degrees (Figure 2).

What types of proof schemes (following Harel and Sowder, 1998) did the participants utilize to convince initially themselves and later to convince others about the pertinence and validity of their results? Table 1 shows a summary of the type of proof schemes used by the three participants.

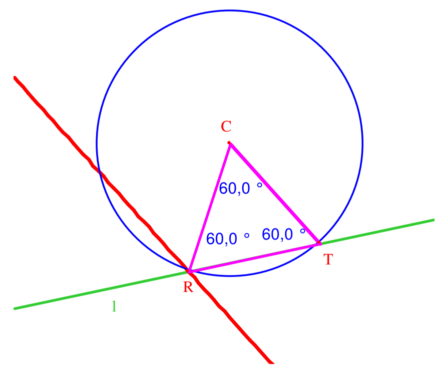

Figure 2. Equilateral triangle.

	Task			
Participant	1	1.1	2	3
Ann	d, g	d	N	d, f
Mary	e, g	e, g	d, e	d
Hugh	e, g	d, e	d, f	d

Type of proof scheme: (a) ritual, (b) authoritarian, (c) symbolic (d) perceptual, (e) inductive, (f) transformational, (g) axiomatic, (N) problem not solved.

Table1: Proof schemes used by the participants

Mary constructed an equilateral triangle by drawing a line L, and a perpendicular to L passing by point C. This perpendicular line cuts line L at M. Then se drew line l_2 and points D_1, D_2 and D_3 such as $MD_1 = D_1D_2 = D_2D_3$. Then drew segment CD_3 and parallel lines to this segment passing by points D_2 and D_1. The latter intersects line MC at T. She drew a circle with center point T and radius TC. This circle intersects line L at points A and B. Here she stated that triangle ABC was equilateral. How did Mary convince herself that the triangle she had constructed was equilateral? Mary, as the other participants, used the software initially to measure the angles in order to verify if they measured each 60 degrees (Figure 3a). When the participants exchanged ideas and discussed their approaches with the whole group, they recognized the importance of providing other type of evidence to show that, in this case that the triangle was equilateral. For example, Mary at the end of the session in her report wrote: CM is perpendicular to L (by construction) and T divides segment CM into a ration 2:1. Let h be equal to TM, then CT = AT = BT = 2h, this is because CT, AT and BT are radii of the same circle (Figure 3b). Triangle AMT is right triangle, therefore, MA = $\sqrt{3}h$; similarly CA = $2\sqrt{3}h$, and CB = $2\sqrt{3}h$. As a consequence triangle ABC is equilateral. A key idea used in Mary's report is the identification of point T (center of the circle). Her construction was based on assuming the existence of the equilateral triangle and to identify its relevant properties. That is, she used the properties to guide her construction.

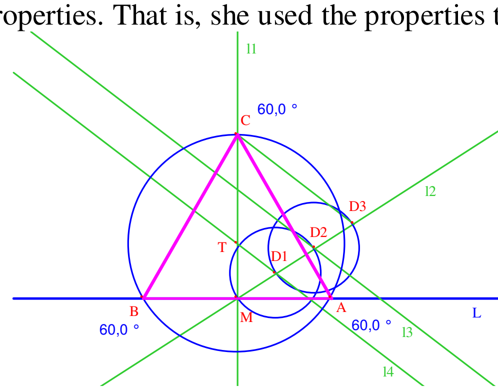

Figure 3a. Constructing an equilateral triangle.

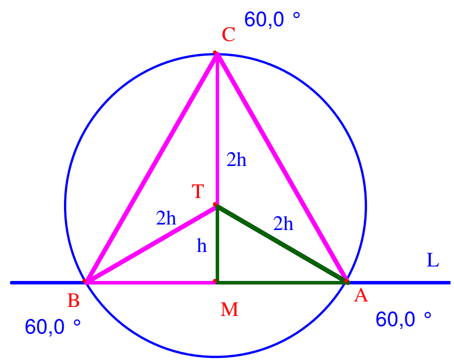

Figure 3b. Providing an argument to validate the construction.

Hugh observed that the locus of the perpendicular bisector when point Q was moved along line l (Figure 1b) seemed to be a parabola. His first strategy to convince himself that the locus was a parabola was to use the software command (conic) to visualize if five points on that locus determined that conic (parabola) (Figure 4a). At this stage, he was convinced that the locus was a parabola; but he was aware that it was important to think of other types of arguments. Later, he chose point P on the locus and drew a perpendicular line to l that passes through point P. This perpendicular line intersects line l at point R. Then, with the use of the software he measured distances PR and PC and observed that for different position of point P the distances were the same (Figure 4b). Here, Hugh assumed that line l and point C were the directrix and focus of the parabola respectively and used the software to verify the definition of this conic.

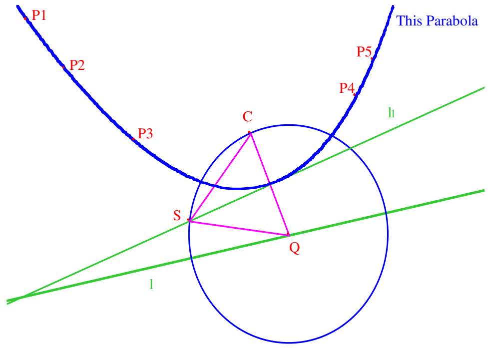

Figure 4a. Perceptual scheme to validate the existence of a parabola.

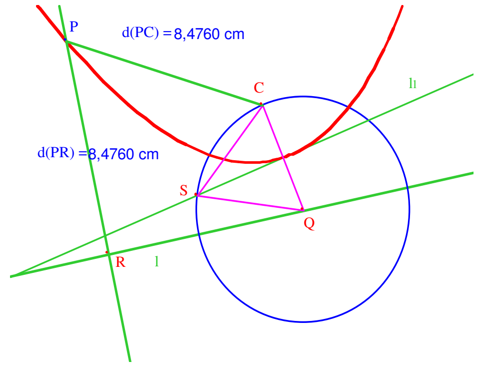

Figure 4b. Empirical validation of the definition of a parabola.

The use of the software also allowed the participants to display transformational proof schemes. For example, Ann approached the task that involved the construction of a tangent circle to two given circles by initially focusing on a partial solution. That is, given the circles with centers at point A and B respectively, she situated points R and Q on each circle. Then she drew lines AR and BQ and observed that for certain positions of these points the lines get intersected at point C. She drew a circle with center at point C and radius CR. This circle is tangent to circle with center at point A (Figure 5a) (partial solution). Ann noticed visually that when point R is moved along the circle there was a point on circle with center B at which the circle with center C is tangent to both circles. To justify this construction, Ann argued: For certain positions of point R the circle with center C does not intersect the circle with center B while for other positions of point R the circle intersect that circle at two points (Figure 5b), then there should be a position for R in which the circle intersects the other at only one point. That is, there must be a position for point R in which the circle with center at C is also tangent to the circle with center at point B.

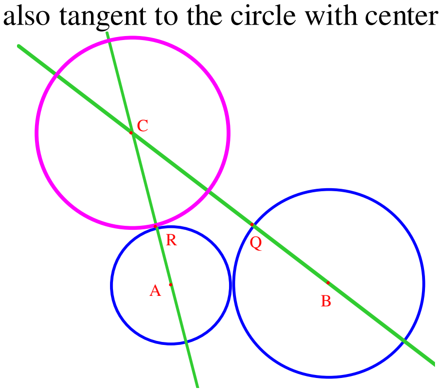

Figure 5a. A partial solution to the problem.

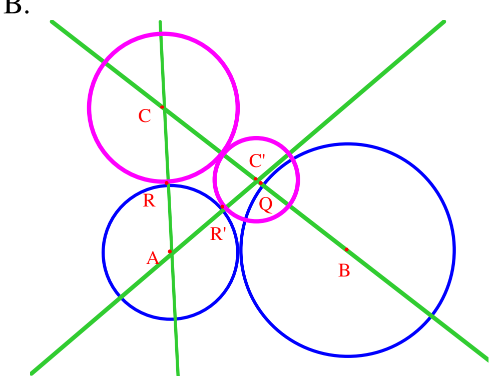

Figure 5b. Dragging point R around circle with center A to find the solution.

Another example of the appearance of a transformational scheme is shown in Hugh's approach to the construction of the square. He constructed a family of rectangles holding the condition that P was one vertex and Q the middle point of the opposite side (Figure 6a). When point N is move along the circle, he observed that one element of that family of rectangles represented the solution of the problem. To justify his method to identify the square, Hugh argued that when point N is moved along the circle, the family of the generated rectangles holds initially that the length of segment PN is less than the length of segment PR

(Figure 6a); however, for certain positions of point N it appears that the length of segment PN is greater than the length of segment PR (Figure 6b). Therefore, there should be the case in which both lengths are equals.

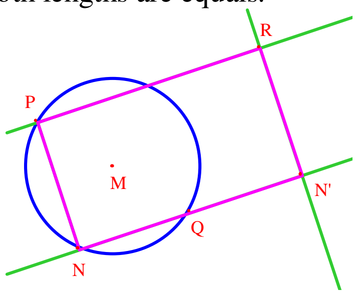

Figure 6a. M is the middle point of segment PQ and N' symmetric to N with respect to Q.

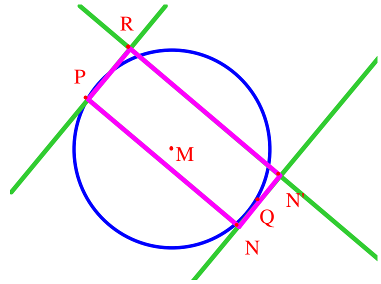

Figure 6b. The length of segment PN is greater than the length of segment PR.

Final Remarks

There is evidence that the use of the software helped the participants construct dynamic representations of mathematical objects that eventually became a source or departure point to formulate questions and problems. What relevant features characterize the participants' process to construct a dynamic representation of the situation? It was observed that the participants started to analyze the situation in terms of geometric properties and translated this information into the construction of objects that eventually could be moved and observed components' behaviors. For example, when Hugh situated point Q on a line and point S be part of a circle, he was aware that when moving point Q on line l, it was important to follow the path left by point S and the perpendicular bisector of segment CQ. Indeed, tracing those loci led them to construct the equilateral triangle and to identify the locus of the perpendicular bisector as a parabola. At this stage, Hugh directed his attention to finding distinct types of arguments to support his finding. Again, the use of the tool was relevant to explore a quantitative approach (measuring distances and angles) to initially verify the properties of those loci.

Although some of the proof schemes identified by Harel and Sowder (1998) seemed to appear in the participants' problem solving approaches, there is evidence that with the use of the tool, the participants can move from visual, empirical and perceptual approaches to more formal or deductive schemes. In addition, the participants' process of posing problems helped them to initially be convinced that the problem or question and associated conjectures were relevant and needed to be explored or supported. As a consequence, it was natural to think of different ways to support their responses.

Finally, the use of the tool seems to enhance problem solving strategies that include (i) assuming the problem solved and then to identify properties to construct a dynamic representation; (ii) representing and solving the problem partially and then examining the representation by moving some elements within the representation to find the complete solution; and (iii) using the tool "locus" to observe the behavior of some elements of the representation to solve the problem or to formulate other questions or problems.

Acknowledgment

This report is based on an ongoing study that is part of the doctoral program in mathematics education at the Center for Research and Advanced Studies (Cinvestav) presented by the first author under the direction of the second author. We acknowledge the support received by Conacyt (reference No 47850), during the development of this research.

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